Groupe de travail sur *The Chromatic Nullstellensatz* à l'Université Sorbonne Paris Nord

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1. **Introduction to the program.**

2. **Deformation theory and spherical Witt vectors.** Give enough context to state and prove [1, Proposition 2.2] providing a definition of the spherical Witt vectors functor $\mathbb{W}$ (cf. [4, §5.2]). Describe the essential image of the spherical Witt vectors functor [1, Lemma 2.11] and provide the necessary context from deformation theory. Discuss examples from [1, §2.2.2], especially the discussion of strict elements from [1, Construction 2.14]. Finally discuss the essential properties of the tilt functor $(-)^\flat$ in [1, §2.3].

3. **Lubin–Tate E-theory.** First, give context by discussing the Goerss–Hopkins–Miller theorem à la Lurie [4, §5]. Then state [1, Theorem 2.38] and sketch the proof. Finally, cover enough background from [?BSY, §3.01] and state [1, Theorem 3.50] and [1, Corollary 3.51].[1]

4. **Detecting nilpotence.** Define the relevant notions of nilpotence and weak rings [1, Definition 4.6,4.11-4.12]. Then state and prove [1, Lemma 4.13]. Discuss the notion of conservative object and state and prove [1, Lemma 4.18]. Discuss the notion of a weakly saturated class of morphisms [1, Definition 4.33] and the small object argument à la Lurie [1, Proposition 4.35]. State [1, Theorem 4.36] and [1, Corollary 4.37] and sketch the proof if time permits.

5. **Examples of nilpotence detection.** Explain criteria for a map f of perfect $\mathbb{F}_p$-algebras to induce a map $\mathbb{W}_{\mathcal{C}}(f)$ that detects nilpotence [1, Theorem 4.47, Lemma 4.48, Theorem 4.49]. Highlight the case when the unit is compact [1, Lemma 4.51, Theorem 4.5.2].

6. **Height zero case and outline of the higher height case.** State [1, Theorem 5.3] and prove the theorem in detail. State [1, §5.2]. State [1, Corollary 5.2] and outline the proof. Explain why the map from the unit in $T(n)$-local spectra to Lubin–Tate E-theory detects nilpotence [2]. State and prove [1, Proposition 5.9]. State [1, Definition 5.10, Proposition 5.11] and explain how they can be used to prove the main theorem.

7. **Higher height case.** Cover [1, §5.3] in as much detail as possible. Cover [1, §5.4] in as much detail as possible.

8. **Algebraic K-theory of Lubin–Tate E-theory and redshift.** Discuss the main points from [3]. Cover the parts [5, §2-4] relevant to algebraic K-theory of Lubin–Tate E-theory. Cover [1, §9] and discuss open questions.

References

[1] Robert Burklund, Tomer M. Schlank, and Allen Yuan, *The Chromatic Nullstellensatz*, arXiv e-prints (July 2022), arXiv:2207.09929, available at 2207.09929.

[1] We plan to blackbox the rest of [1, §3] in this seminar.

[2] Ethan S. Devinatz, Michael J. Hopkins, and Jeffrey H. Smith, *Nilpotence and stable homotopy theory. I*, Ann. Math. (2) **128** (1988), no. 2, 207–241 (English).

[3] Lars Hesselholt and Thomas Nikolaus, *Topological cyclic homology*, Handbook of homotopy theory, 2019.

[4] Jacob Lurie, *Elliptic cohomology II: Orientations*, 2018. https://www.math.ias.edu/~lurie/papers/Elliptic-II.pdf.

[5] Allen Yuan, *Examples of chromatic redshift in algebraic K-theory*, arXiv e-prints (November 2021), arXiv:2111.10837, available at 2111.10837.